

Gearbox Design Project Report

Group Number: 10

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(Names are color-coded in respect to work completed by each member)

1. Objective

The goal of this project is to design and analyze a non-reverted two-stage gearbox system for efficient power transmission. The design incorporates stress analysis and safety factor evaluation for gears, shafts, bearings, and keys to meet reliability and durability requirements.

Project Specifications:

- Transmitted Power: 4 HP
- Input Speed: 1500 RPM
- Output Speed: 30-160 RPM
- Desired Life: 55,000 Hours
- Material: AISI 1095 Q&T
- Operating Temperature: 50–80°F

2. Method

2.1 Gears

The gearbox shown below is a non-reverted gearbox, characterized by its configuration where the input and output shafts are on different axes. In this arrangement, power transmission occurs through intermediate gears (Gear 1, 2, and 3), which allows the input and output to rotate at different speeds and directions.

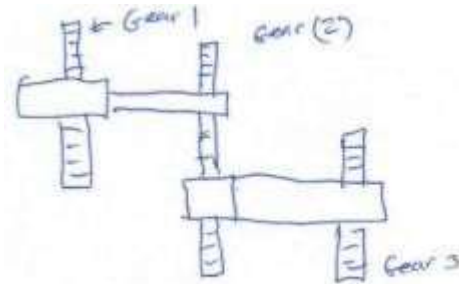


Figure 1: Non-Reverted Gear Box

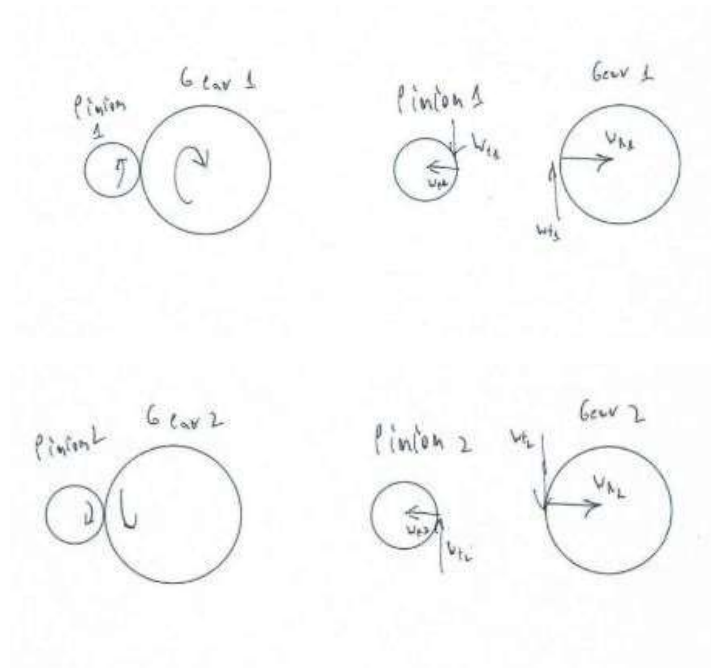


Figure 2: Force diagram on each gear

Design process:

1. Determine gear ratios based on input and output speeds.
2. Calculate bending and contact stresses using AGMA equations.
3. Select materials with adequate hardness and strength.
4. Validate safety factors for both bending and contact stresses.

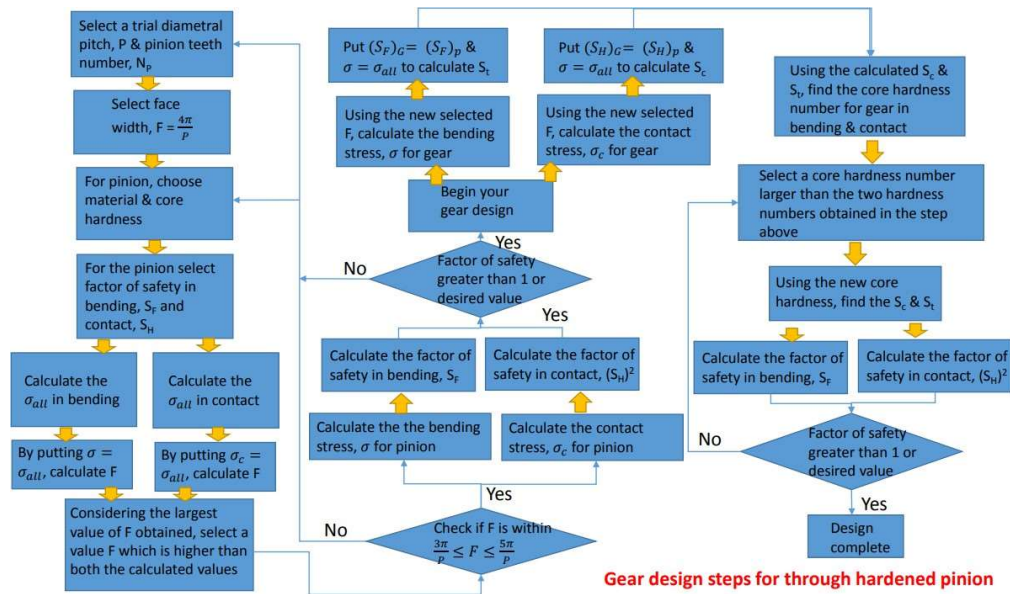


Figure 3: Flow Chart for Gears

Major equations:

Gear bending stress equation Eq. (14-15)

$$d_p = \frac{N_p}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W' = \frac{33\,000 H}{V}$$

$$\sigma = W' K_t K_m K_b \frac{P_d}{F} \frac{K_m K_b}{J}$$

1 [or Eq. (a), Sec. 14-10]; p. 751
Eq. (14-30); p. 751
Eq. (14-40); p. 756
Fig. 14-6; p. 745
Eq. (14-27); p. 748

Table below

Gear bending endurance strength equation Eq. (14-17)

$$\sigma_{all} = \frac{S_t}{S_F} \frac{Y_N}{K_T K_R}$$

$0.99(S_t)_{10^7}$ Tables 14-3, 14-4; pp. 740, 741
Fig. 14-14; p. 755
Table 14-10, Eq. (14-38); pp. 756, 755
1 if $T < 250^\circ\text{F}$

Bending factor of safety Eq. (14-41)

$$S_F = \frac{S_t Y_N / (K_T K_R)}{\sigma}$$

Gear contact stress equation Eq. (14-16)

$$d_p = \frac{N_p}{P_d}$$

$$V = \frac{\pi d n}{12}$$

$$W' = \frac{33\,000 H}{V}$$

$$\sigma_c = C_p \left(W' K_t K_m K_b \frac{P_d}{F} \frac{K_m K_b}{J} \right)^{1/2}$$

1 [or Eq. (a), Sec. 14-10]; p. 751
Eq. (14-30); p. 751
Eq. (14-23); p. 747
Eq. (14-27); p. 748
Eq. (14-13), Table 14-8; pp. 736, 749
Table below

Gear contact endurance strength Eq. (14-18)

$$\sigma_{c,all} = \frac{S_c Z_N C_H}{S_H K_T K_R}$$

$0.99(S_c)_{10^7}$ Tables 14-6, 14-7; pp. 743, 744
Fig. 14-15; p. 755
Section 14-12, gear only; pp. 753, 754
Table 14-10, Eq. (14-38); pp. 756, 755
1 if $T < 250^\circ\text{F}$

Wear factor of safety Eq. (14-42)

$$S_H = \frac{S_c Z_N C_H / (K_T K_R)}{\sigma_c}$$

Gear only

2.2 Shafts

Design process:

1. Analyze the critical locations, such as the shoulders and keyseats.
2. Apply the criteria for fatigue and yielding.
3. Select the diameters to meet safety factor requirements.

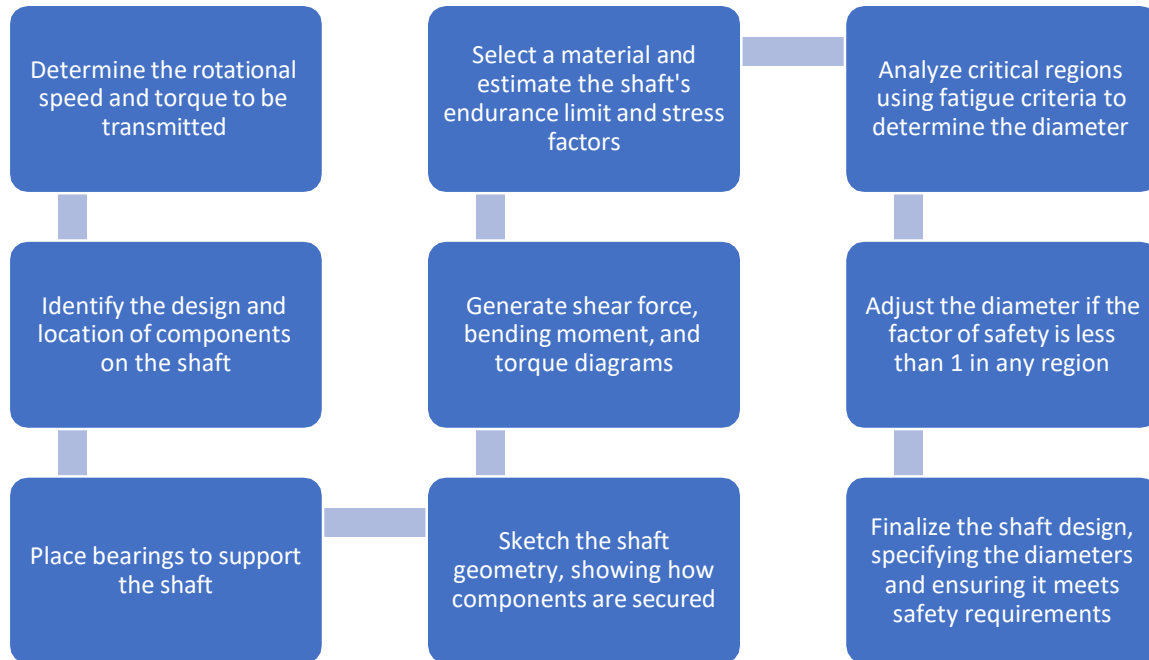


Figure 4: Flow Chart for Shafts

Major equations:

$$\sigma'_a = (\sigma_a^2 + 3\tau_a^2)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left(\frac{16K_{fs} T_a}{\pi d^3} \right)^2 \right]^{1/2}$$
$$\sigma'_m = (\sigma_m^2 + 3\tau_m^2)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left(\frac{16K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2}$$
$$d = \left(\frac{16n}{\pi} \left\{ \frac{1}{S_e} [4(K_f M_a)^2 + 3(K_{fs} T_a)^2]^{1/2} + \frac{1}{S_{ut}} [4(K_f M_m)^2 + 3(K_{fs} T_m)^2]^{1/2} \right\} \right)^{1/3}$$

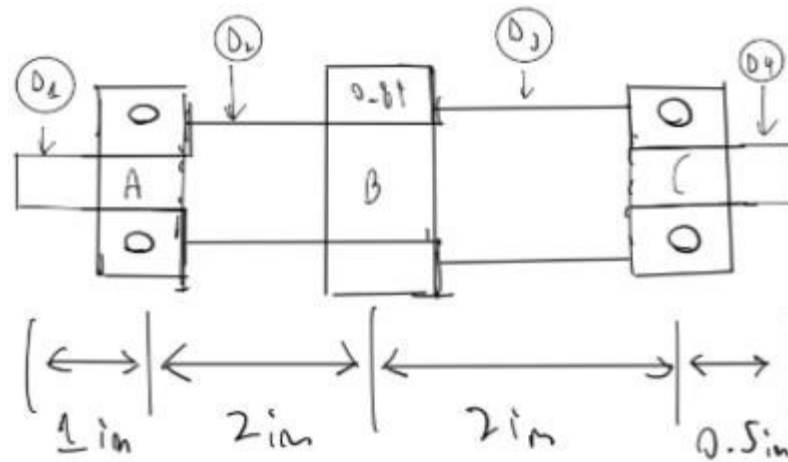
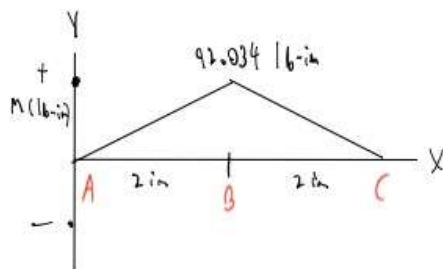


Figure 5: Shaft 1 Layout

Bending moment diagram in Y-axis:



Shear force diagram in Y-axis:

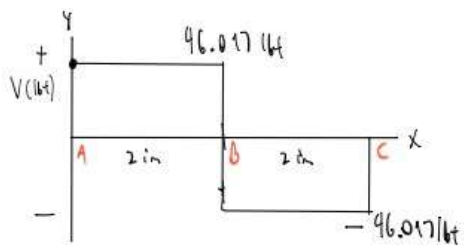


Figure 6: Bending Moment and Shear Force Diagram for Shaft 1

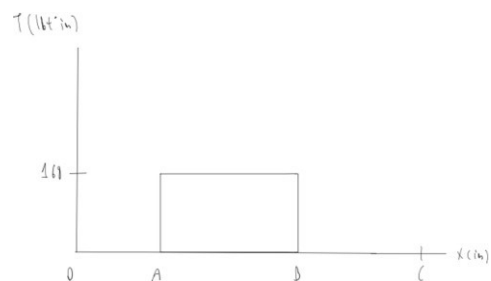


Figure 7: Torque Diagram for Shaft 1

2.3 Bearings

Selection Process:

To select a bearing, obtain the Weibull parameters and rated life from the manufacturer. Calculate the life multiple, application factor, and design load. Determine the load versus life exponent and calculate the required C10 load rating using standard equations. Choose a bearing from the catalog that meets or exceeds this C10. Estimate the bearing's reliability using its actual C10 and account for reliability across components. Finally, consider combined radial and thrust loads and any variable loading to ensure the bearing meets all requirements.

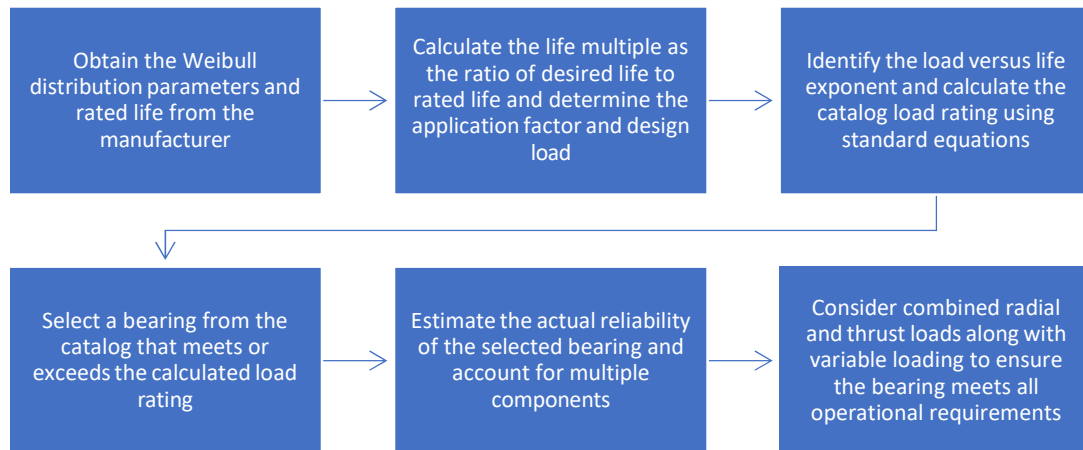


Figure 8: Flow Chart for Bearings

Major Equations:

$$C_{10} = F_R = F_D \left(\frac{L_D}{L_R} \right)^{1/a} = F_D \left(\frac{\mathcal{L}_D n_D 60}{\mathcal{L}_R n_R 60} \right)^{1/a}$$

Calculations for the bearings are in the appendix*

2.4 Keys

Design Process:

Key design involves selecting the key size, type, and material based on shaft diameter and strength. The length is calculated for shear and bearing stresses, checked against the hub or face width, and adjusted as needed.

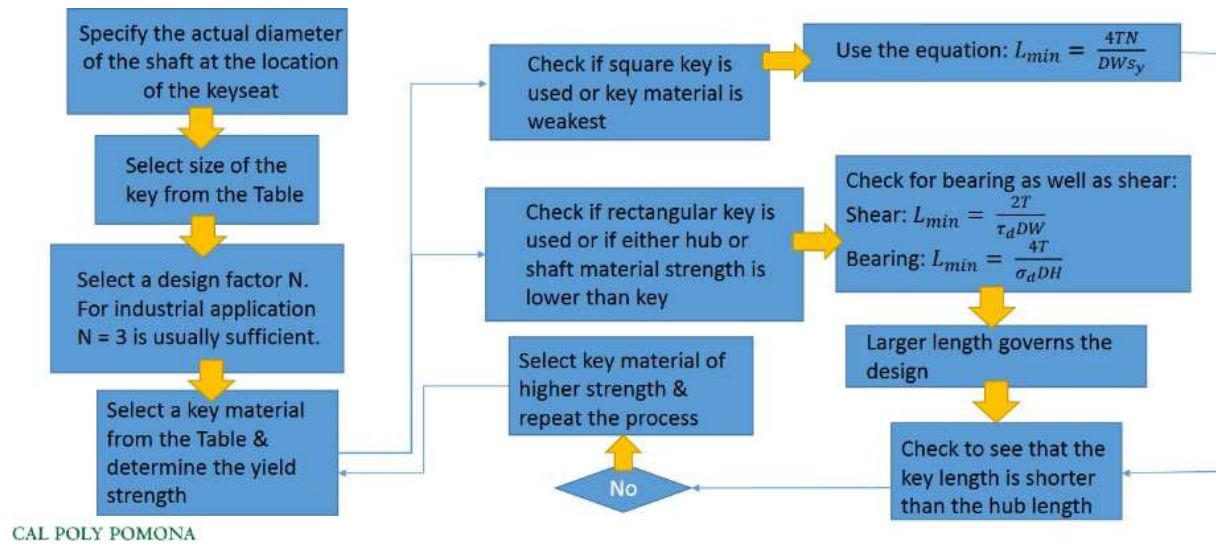


Figure 9: Flow chart for Keys

Major equations:

- Shear force acting on the key:

$$F = \frac{T}{D/2}$$

- Area of shear: $A_s = WL$

- Shear stress:

$$\tau = \frac{F}{A_s} = \frac{T}{\left(\frac{D}{2}\right)(WL)} = \frac{2T}{DWL}$$

Calculations for the keys are in the appendix*

3. Results

3.1 Gears

Analysis: Gear ratios in each stage

- Stage 1: Gear Ratio = 5 Stage 2: Gear Ratio = 7

The desired safety factors for bending (1.5) and contact stresses (1.2) were met by Pinion 1 and are acceptable, ensuring reliability. Even with an increased safety factor for bending it is more susceptible to failing before contact stress.

Table 3.1: Specification, force, stress and safety factors for pinions and gears

	Diametral Pitch (teeth/in)	Pitch Diameter (in)	No. of Teeth	RPM	W ^t (lbf)	Face Width (in)	Bending Stress (psi)	Contact Stress (psi)	S _F	S _H
First Stage										
Pinion	15	1.33	20	1500	252.7	0.84	27927	136654	1.6	1.2
Gear										
Second Stage										
Pinion										
Gear										

Table 3.1: This table provides key design and operational parameters for the pinion and gears. It includes specifications such as diametral pitch, pitch diameter, number of teeth, and rotational speed. It also lists applied forces, bending and contact stresses, and safety factors for bending and contact stress. This data ensures the pinion and gears can handle the operational loads safely and efficiently while maintaining durability.

Table 3.2: Material properties for pinions and gears

	Brinell Hardness Number	S _t (kpsi)	S _c (kpsi)
Stage 1			
Pinion	375	41.788	149.850
Gear			
Stage 2			
Pinion			
Gear			

Table 3.2: This table details the material characteristics of the pinion and gears, including Brinell hardness, tensile strength, and contact stress limits. These properties ensure the materials can withstand the applied stresses and maintain performance under operational conditions and is critical to prevent failure.

3.2 Shafts

Analysis:

Critical Shaft Diameters

- Shoulder at A: 0.52 in Keyseat at B: 0.66 in

Safety factors:

- Fatigue (A): 4.7 Fatigue (B): 5.19 Yielding (A): 8.69 Yielding (B): 17.92

These factors of safety are acceptable, but they seem steep. It is believed that an error was made in one of the variables during the calculation process as they are unreasonably high.

Table 3.3: Summary of diameters for shafts

	D1 (in)	D2 (in)	D3 (in)	D4 (in)	D5 (in)
Shaft 1	0.55	0.70	0.85	0.55	N/A
Shaft 2					
Shaft 3	.55	.75	.80	.60	0

Table 3.3: Lists the diameters of the shaft at various critical sections. These measurements are essential for ensuring proper fit with components like bearings and gears, as well as maintaining structural integrity under operational loads.

Table 3.4: Summary of critical quantities for shaft design

	Bending Moment (lbf-in)	Torque (lbf-in)	K _f	K _{fs}	Fatigue Safety Factor
Shaft 1					
Shoulder at A	0	168	1.44	1.32	7.71
Keyseat at B	289.08	168	1.67	2.22	5.19
Shoulder at B	289.08	168	1.42	1.27	4.71
Shaft 2					
Shoulder at B					
Keyseat at B					
Shoulder at C					
Keyseat at C					
Shaft 3					
Shoulder at B	0	158	1.41	1.52	7.17
Keyseat at B	281.1	158	1.67	2.22	5.19
Shoulder at C	281.1	158	1.50	1.31	4.51

Table 3.4: This table provides information on the forces acting on the shaft, including bending moments and torque, along with stress concentration factors. It also includes fatigue safety factors to evaluate the shaft's durability under repeated loading.

Table 3.5: Summary of yielding factor of safety

	Maximum Von Mises Stress (kpsi)	Yielding Safety Factor
Shaft 1		
Shoulder at A	13.52	8.69
Keyseat at B	6.58	17.93
Shoulder at B	6.58	17.93
Shaft 2		
Shoulder at B		
Keyseat at B		
Shoulder at C		
Keyseat at C		
Shaft 3		
Shoulder at B	9.15	8.09
Keyseat at B	6.18	15.53
Shoulder at C	6.08	15.53

Table 3.5: This table highlights the maximum stress experienced by the shaft and the corresponding yielding safety factors. It ensures the shaft can withstand operational stresses without permanent deformation or failure.

Table 3.6: Material properties of shafts

	Material	Yield strength (kpsi)	Ultimate Tensile Strength (kpsi)
Shaft 1	AISI 1095 Q&T at 315C	118	183
Shaft 2			
Shaft 3	AISI 1095	118	183

Table 3.6: This table describes the shaft material, including its strength properties. It ensures the material can withstand the stresses and loads expected during operation.

3.3 Bearings

Analysis:

These bearings had reliability of 90%. For Shaft 1 the number of expected revolutions was 450,000,000 (5,000 hours at 1,500 RPM). The realized reliability using the selected C10 value is nearly 100%, meaning failure is highly unlikely within the desired life. The SKF 61817 bearing has a C10 of 4,384 lbf, which is significantly higher than the calculated requirement. This provides a safety margin and ensures the bearing can handle the load comfortably for a longer life.

Table 3.7: Summary of critical quantities for bearing selection

	Type	Bore diameter (inches)	C ₁₀ (lbf)	C ₀ (lbf)	F _D (lbf)
Shaft 1					
Right Bearing	SKF 61817	3.346	4,384	3,732	252.7
Left Bearing	SKF 61817	3.346	4,384	3,732	252.7
Shaft 2					
Right Bearing					
Left Bearing					
Shaft 3					
Right Bearing	skf61817	3.173	4.192	4,366	249.4
Left Bearing	skf61817	3.173	4.192	4,366	249.4

Table 3.7: Provides the bearing type, which specifies the model and design; the bore diameter, which is the inner diameter of the bearing that fits onto the shaft. The dynamic load rating, which indicates the maximum radial load the bearing can sustain for a specified life. The static load rating, which represents the maximum radial load the bearing can endure without permanent deformation, and the applied load which is the actual load acting on the bearing during operation.

3.4 Keys

Analysis:

The selected key for Shaft 1 and Pinion 1 has a square cross-section as the width and height are equal. A factor of safety of 3 was used to calculate the key length to ensure safe operation under applied stresses. Since the gear does not have a hub, the key length was adjusted from 35.37 in to 0.84 in to match the gear face width, ensuring it fits properly and does not overhang.

Table 3.8: Summary of critical quantities for key design

	Key width (inches)	Key height (inches)	Key length (inches)	Key material	Yield strength (kpsi)
Shaft 1 & Pinion in 1 st stage:	0.125	0.125	0.84	1095 Steel	83
Shaft 2 & Gear in 1 st stage					
Shaft 2 & Pinion in 2 nd stage					
Shaft 3 & Gear in 2 nd stage	.120	.120	.81	1095 steel	83

4. Conclusion

This project successfully designed a two-stage non-reverted gearbox system. The analysis ensured optimal safety factors for gears, shafts, bearings, and keys, meeting the desired specifications. The gears were designed to transmit power efficiently, with dimensions and materials selected to withstand bending and contact stresses while achieving the desired speed ratio. Shafts were designed to effectively transmit torque and support applied forces, while bearings were selected to handle both radial and axial loads, ensuring reliability over the desired lifespan. The keys were dimensioned and made from suitable materials to resist failure under shear and crushing stresses, maintaining compatibility with the gear hubs and other requirements. Overall, the gearbox design fulfills the desired operational goals with good performance and reliability.

References

Abedin, Farhana. *Gear Design Flow Chart*. PowerPoint presentation.

Abedin, Farhana. *Gear Design: Spur Gear*. PowerPoint presentation.

Abedin, Farhana. *Shaft Design*. PowerPoint presentation.

Abedin, Farhana. *Design of Square/Rectangular Keys*. PowerPoint presentation.

Abedin, Farhana. *Rolling Contact Bearings*. PowerPoint presentation.

Budynas, R. G., and Nisbett, J. K. *Shigley's Mechanical Engineering Design*. 10th edition, McGraw-Hill Education, 2015.

Appendix

Calculations for bearings on Shaft 1:

Bearing Calculations

$$X_D = \frac{L_D}{L_R} \rightarrow \frac{45 \times 10^7}{1 \times 10^6} = 450$$

$$C_{10} = F_D X_D^{1/a} \rightarrow (252.7)(450)^{1/3} = \underline{1,936.5 \text{ lbf}}$$

SKF 61817 chosen so $C_{10} > 1,936.5 \text{ lbf}$ ← catalog bearing

$$L_{\text{actual}} = 10^6 \left(\frac{C_{10}}{F_D} \right)^3 \rightarrow 10^6 \left(\frac{1936.5}{252.7} \right)^3 = 5.23 \times 10^6 \text{ rev}$$

$$\text{Reliability} = R = e^{-\left(\frac{L_D}{L_R}\right)^3} \rightarrow R = e^{-\left(\frac{45 \times 10^7}{5.23 \times 10^6}\right)^3} = 0.9994 = \underline{99.94\%}$$

Calculations for keys on Shaft 1:

Square key Calculations

$$\text{Key width/height} = \frac{D_1}{4} = \frac{0.55 \text{ in}}{4} = 0.1375 \text{ in}$$

table for diameter $\downarrow$
0.125 in
 width/height

$$\tau_{ol} = 0.5 \frac{y_s}{n} = 0.5 \left(\frac{83 \text{ ksi}}{3} \right) = 13.83$$

$$\sigma_d = \frac{y_s}{n} = \frac{83 \text{ ksi}}{3} = 27.67$$

$$L_{\text{min (shear)}} = \frac{2T}{\tau_{ol} w D} = \frac{(2)(168)}{(13.83)(0.125)(0.55)} = 35.37 \text{ in}$$

$$L_{\text{min (compression)}} = \frac{2T}{\sigma_d w D} = \frac{(4)(168)}{(27.67)(0.125)(0.55)} = 35.37 \text{ in}$$

35.37 in $\rightarrow$ 0.84 in
 limited to $\rightarrow$ Pinion 1 Face width
 Length